

Prediction of Long-Term Creep of Composites from Doubly-Shifted Polymer Creep Data

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Abstract¹

Environmental effects on the creep response of composites are of great interest because these materials are very sensitive to temperature and other environmental conditions. Short and long-term creep effects have been studied for composites under various environmental conditions, but most studies rely on experimental testing of particular laminates. Here, a novel micro-mechanical/laminate model is developed to predict the creep response of laminated polymer matrix composites from matrix creep data. Furthermore, a novel two-parameter superposition method is used to predict the long-term response of the polymer matrix as a function of two environmental parameters: moisture and temperature. The procedure used to compute the two shift factors is described. The resulting master curve can be used to predict the long-term properties of the matrix and for any temperature and humidity conditions. Once the matrix behavior is known, the micro-mechanical/laminate model allows for accurate prediction of long-term properties of arbitrary laminates for the same conditions.

List of Symbols

$\hat{\alpha}$	Inplane compliance matrix
$\hat{\varepsilon}$	Inplane strain resultant array
λ_m, λ_f	Lame constant of the matrix and fiber, respectively
μ_m, μ_f	Shear modulus of the matrix and fiber, respectively
ν_m, ν_f	Poisson's ratio of the matrix and fiber, respectively
ϕ, ϕ_0	Relative and reference relative humidity
Θ	2D rotation tensor [26,(5.35)]
$\overline{\Theta}^{(4)}$	3D rotation tensor [24,(1.40)]
a_ϕ	Humidity horizontal shift factor
a_H	Horizontal shift factor
a_T	Temperature horizontal shift factor
a_V	Vertical shift factor
b_ϕ	Humidity vertical shift factor

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b_T	Temperature vertical shift factor
k	Layer number
t	time
$t(k)$	Layer thickness
\hat{A}	Inplane stiffness matrix
\hat{C}	Transversely isotropic relaxation tensor in the Carson domain
E_m, E_f	Relaxation modulus of the matrix and fiber, respectively
$J(t)$	Creep compliance
J_0	Initial creep compliance
J', n	Parameters in the power law model
K, C	Parameters in the Maxwell model
\tilde{L}	Relaxation tensor in the Laplace domain
\hat{L}	Relaxation tensor in the Carson domain
\hat{N}	Inplane stress resultant array
N_L	Number of layers in the laminate
\hat{Q}	2D stiffness of a unidirectional layer
\hat{Q}^{CSM}	2D stiffness of a continuous strand mat layer
T	Temperature
V_f	Fiber volume fraction

1. Introduction

Polymer Matrix Composites (PMC) display important viscoelastic behavior, which is sensitive to environmental effects (temperature, humidity [1], radiation [2], etc.). Short and long term creep effects have been studied experimentally, but very few models have been developed to predict long-term creep of laminated composite materials from short-term data. This work presents an approximate methodology to predict long-term creep of composite laminates from short-term constituent data. Two contributions are described. First, a doubly-shifted superposition method is developed to account for temperature and moisture conditions. Second a combined micro-mechanics/laminate model allows us to predict laminate creep from constituent data. The predictions are validated with experimental data for relatively short times for which aging effects can be neglected. Aging effects [3], [4] and deviations from a thermorheologically complex material (TCM) behavior [5], [6] are not considered in this study although they may effect the accuracy of the predictions for very long times. Because our intention is to develop a simple methodology that can be used for preliminary design, every effort was made to arrive at the simplest model. Experimental data is presented for neat resin (matrix) and laminated composites including continuous strand mat, unidirectional, and bidirectional stitched fabric layers. Neat resin samples were tested to obtain the matrix creep properties directly, without relying on back calculation using any micro-mechanics equations. Besides being economical, this method has the advantage that various matrices can be experimentally qualified for creep response under various environmental conditions (temperature, moisture, radiation) before fabricating any composite samples.

The correspondence principle [7] offers a powerful tool for modeling linearly viscoelastic materials; that is, when the viscoelastic behavior can be assumed to be independent of the stress level. Most structures are likely to operate in a known and relatively narrow range of stress values. The stress range is known because the sizing of members, and thus the level of stress is determined to satisfy requirements such as maximum deflections, fatigue limits, notched strength, and compression after impact, etc. Therefore, in preliminary design, it is convenient to use linear viscoelasticity at the known stress range rather than using complex nonlinear models [1], [8], [9], [10]. Using the correspondence principle, most analytical tools developed for elastic materials can be used by taking a Laplace transform. This excludes iterative techniques (e.g., the self consistent method) and formulas containing empirical factors (e.g. Halpin-Tsai equations [11]) because the dependence of empirical factors as a function of time is unknown. In contrast, we use asymptotically exact micromechanics [12] without empirical correction factors, yielding analytical expressions, which facilitate the evaluation of the Laplace transform.

The creep behavior of any linearly viscoelastic material can be represented by empirical models such as the power law, Maxwell, Ideal solid, etc. [13], [14]. These empirical models simply curve fit the experimental creep data. Once the creep behavior (shear, longitudinal, and transverse) of the composite is known from experiments [15], either at the layer or laminate level, several numerical methods exist to integrate the viscoelastic equations [16], [17], [18], [19], [20]. A problem encountered with these proposed models is that they rely on experimental data for a particular laminate. This means that an expensive experimental program needs to be completed before attempting the design of the structure. The experimental program is very expensive for composites because the need to consider various fibers and resin types, several values of fiber volume fraction, and a number of temperature and moisture conditions, etc. Furthermore, several types of tests are necessary to evaluate the creep behavior in shear as well as longitudinal and transverse directions [21]. The proposed procedure allows us to predict all the components of the creep compliance tensor from the elastic properties of the fibers and the matrix creep data only.

2. Experimental Program

Thirty-six different tests were performed, each with four replicates, for a total of 144 samples. In addition to neat resin samples, three laminates described in Table 1-2 were tested. The four materials were tested in nine conditions, three temperatures (-12.2, 21.1, 65.6 °C) and three relative humidity (0%, 12%, 80% RH). Temperature and moisture-concentration equilibrium were reached before testing on all samples. The laminates were fabricated using vacuum assisted resin transfer molding (VARTM), also known as SCRIMP. The matrix was Derakane 411-350 with Cumene Hydroperoxide (CHP) used as catalyst and (CoNap) Cobalt Naphthenate (CoNap) used as a promoter. The fabrics were supplied by Brunswick Technology Inc. The composites were fabricated by Hard Core DuPont using Derakane resin supplied by Dow Chemical.

The creep response at constant load is found by using a fixture that applies a dead load to a specimen placed inside a Cincinnati Sub Zero ZH-32 environmental chamber. The chamber has a temperature range of -40 °C to 190.6 °C and a humidity range of 0 % RH to 99 % RH. The specimens are loaded through pins which allows for thermal

expansion of the system and for the specimen to be aligned properly. The specimen is dog boned to ensure that the maximum creep region will be in the center of the specimen. The dimensions were selected to prevent creep or failure at the grip region for the neat matrix samples [22]. The composite specimens were fabricated with the same dimensions, which provided more than sufficient strength in the grip region.

A Micro Measurements CEA-13-250UN-120 strain gage was mounted on the sample. Micro Measurements M-bond 200 adhesive was used to fasten the gage to the specimens. An RTV coating (Dow Corning 3140 RTV) was applied over to protect the gage from moisture. The system has a temperature range of $-73.3\text{ }^{\circ}\text{C}$ to $204.4\text{ }^{\circ}\text{C}$ up to 100% RH for continuous use in static measurements and has a strain limit of 5%. The strain gage was wired for temperature and apparent thermal strain compensation [23]. Finally, the load was applied to the specimen by a crank mechanism, which allows for instantaneous loading.

The strength and elastic modulus were determined a priori by a quasi-static test in a universal testing system. During creep testing, the stress level was kept within the linearly elastic range of the stress-strain response. The test were run until a well-defined secondary creep region developed. A time period of four hours proved to be sufficient for all temperature and humidity conditions included in the experimental program.

The matrix was tested at a constant stress level approximately one quarter of the ultimate strength (11 MPa). Two sets of four specimens were tested at each environmental condition.

The creep response of the three laminates (Table 1-2) experience less initial strain than the matrix, and the secondary creep rates of the laminates are lower in value compared to the matrix. The matrix was tested for 4 hours and the laminates for 22 hours at approximately one quarter of the ultimate strength of each laminate (9 MPa and 29.6 MPa respectively).

3. Empirical Fit of Creep Data

The simplest method for representing creep compliance data is by using a power law, which for linear viscoelastic materials is

$$J(t) - J_0 = J' t^n \quad (1)$$

where J_0 represents the initial compliance at $t = 0$, J' and n are fitted to the data by logarithmic linear regression. However, the most common representation used in engineering design is the Maxwell model, which has two parameters, the initial modified compliance $1/K$ and the creep rate $1/C$. The Maxwell model can be illustrated as a single series spring-dashpot system. The mathematical representation is given by

$$J(t) = \frac{1}{K} + \frac{t}{C} \quad (2)$$

under constant applied stress σ_0 . The Maxwell model does not fit well the primary creep region, but represents well the secondary creep region. Accurate modeling of the primary creep region is often not necessary in structural design because these effects occur over a short period of time compared to the service life of the structure. The

parameter $1/C$ is the creep rate of the material and $1/K$ is the modified initial compliance. This model has been primarily used to describe metallic materials which display straight line secondary creep regions over very long periods of time. The use of this model to represent the response of polymer materials has been controversial, because a power law better represents the creep of polymers for relatively short times. For structural design, however, the main interest is on the response for long periods of time. Experimental data for both neat matrix and composites suggest that the Maxwell model provides a good representation for long times. Because of its simplicity compared to the power law, the Maxwell model is routinely used in design. The Maxwell-Voigt 4-parameter model can be used if the primary creep region needs to be modeled accurately. For long values of time, the 4-parameter model reduces to the 2-parameter model. The ideal-solid model [14] can be used if long-term data is available to ascertain the asymptotic zero strain-rate at long times inherent to the model.

4. Doubly-shifted superposition

The polymer is modeled as a thermorheologically complex material (TCM), with two time-dependent shift factors: horizontal and vertical (a_H and a_V). A modified power law is proposed to represent the viscoelastic response of polymer matrices at various temperature and humidity conditions

$$J(t) - J_0(T, \phi) = J' \frac{1}{a_V} \left[\frac{t}{a_H} \right]^n \quad (3)$$

where $J_0(T, \phi)$ is the initial compliance as a function of temperature and relative humidity, J' and n are constants over the full range of temperature T and relative humidity ϕ . The vertical and horizontal shift factors are assumed separable into temperature and humidity components as follows

$$\begin{aligned} a_H &= a_T(T) a_\phi(\phi) \\ a_V &= b_T(T) b_\phi(\phi) \end{aligned} \quad (4)$$

Values for the temperature and humidity components of the shift factors were found for the nine environmental conditions tested, using two empirical models: Maxwell and Power Law, and four materials: neat resin and three laminates described in Tables 1-2.

All values of initial compliance $J_0(T, \phi)$ are known from the static tests at $t = 0$. The initial compliance J_0 was found from a static test at each temperature (reported in Fig. 1) or as the ordinate at the origin ($1/K$ in (2)) as a function of temperature and relative humidity, as reported in Fig. 2.

To solve for the remaining variables, the first step is to choose a reference temperature and relative humidity (21.1 C and 12% in this paper). At the reference state, all shift factors are equal to one. Then, the experimental data is fitted by (3) with $a_T = a_\phi = b_T = b_\phi = 1$ to obtain the parameters J' and n . A linear regression is performed using on the log of (1) to obtain the parameters J' and n for the matrix at the three environmental conditions. The quality of the approximation achieved with the

Maxwell fit is illustrated in Fig. 3.

In the second step, the parameters J' and n are kept fixed while the experimental creep compliance data at various temperatures and reference relative humidity ϕ_{ref} is used to find the temperature component of the horizontal and vertical shift factors a_T and b_T at each temperature for which data is available. Therefore, the values of $a_T(T)$ and $b_T(T)$ are found by minimizing the square of the error while $a_\phi = b_\phi = 1$; that is while the relative humidity is kept at its reference value. In this work, the temperature dependency was determined by using the data for various temperatures but at reference RH of 12% (Fig. 4-5) and reported in Table 3.

Finally, the remaining data (for $\phi \neq \phi_{ref}$) is used to find the values of a_ϕ and b_ϕ by minimizing the error while keeping $a_T(T)$ and $b_T(T)$ unchanged. Then, the moisture dependency was found using the data at other values of RH. The combined shift factors are shown in Fig. 6-7 and reported in Table 4.

The experimental data confirms that the matrix and composite shift factors are identical. Since the carbon fibers are elastic, they effect only the magnitude of the creep compliance, not the temperature and moisture dependency. That is, the combined horizontal and vertical shift factors a_H and a_V are equal for the matrix and for all the laminates analyzed (Table 3-4). Furthermore, the experimental data confirms that the shift factors are independent of the empirical model used to fit the compliance data (Maxwell or Power Law). Therefore, if the master curve and the shift factor plot for the matrix is know, any laminate can be analyzed at any temperature and relative humidity condition. Still, the values of J_0 , J' , and n need to be predicted for the composite in terms of fiber and matrix properties, as explained in the next section.

5. Laminate Model

Using the correspondence principle for linearly viscoelastic materials, the micro-mechanics formulas developed for elastic materials can be used directly in the Laplace domain, provided the formulas do not contain empirical parameters nor they require iteration [24]. The equations are easier to manipulate using the Carson transform (indicated by \hat{L}) instead, which is related to the Laplace transform (indicated by \tilde{L}) by

$$\hat{L}^*[f(t)] = s\tilde{L}[f(t)] \quad (5)$$

where $f(t)$ is any function of time.

The components of the relaxation tensor in the Carson domain for a unidirectional composite with periodically arranged fibers given in [12], as follows

$$\hat{L}_{11}(s) = \hat{\lambda}_m + 2\hat{\mu}_m - V_f \left[\frac{S_3^2}{\hat{\mu}_m} - \frac{2S_6S_3}{\hat{\mu}_m g} - \frac{aS_3}{\hat{\mu}_m c} + \frac{S_6^2 - S_7^2}{\hat{\mu}_m g^2} + \frac{aS_6 + bS_7}{\hat{\mu}_m gc} + \frac{a^2 - b^2}{4c^2} \right] / D$$

$$\begin{aligned}
\hat{L}_{12}(s) &= \hat{\lambda}_m + V_f b \left[\frac{S_3}{2c\hat{\mu}_m} - \frac{S_6 - S_7}{2c\hat{\mu}_m g} - \frac{a+b}{4c^2} \right] / D \\
\hat{L}_{23}(s) &= \hat{\lambda}_m + V_f \left[\frac{aS_7}{2\hat{\mu}_m gc} - \frac{ba+b^2}{4c^2} \right] / D \\
\hat{L}_{22}(s) &= \hat{\lambda}_m + 2\hat{\mu}_m - V_f \left[-\frac{aS_3}{2\hat{\mu}_m c} + \frac{aS_6}{2\hat{\mu}_m gc} + \frac{a^2-b^2}{4c^2} \right] / D \\
\hat{L}_{44}(s) &= \hat{\mu}_m - V_f \left[-\frac{2S_3}{\hat{\mu}_m} + (\hat{\mu}_m - \mu_f)^{-1} + \frac{4S_7}{\hat{\mu}_m(2-2\nu_m)} \right]^{-1} \\
\hat{L}_{66}(s) &= \hat{\mu}_m - V_f \left[-\frac{S_3}{\hat{\mu}_m} + (\hat{\mu}_m - \mu_f)^{-1} \right]^{-1}, \tag{6}
\end{aligned}$$

where

$$\begin{aligned}
D &= \frac{aS_3^2}{2\hat{\mu}_m^2 c} - \frac{aS_6 S_3}{\hat{\mu}_m^2 gc} + \frac{a(S_6^2 - S_7^2)}{2\hat{\mu}_m^2 g^2 c} + \\
&\frac{S_3(b^2 - a^2)}{2\hat{\mu}_m c^2} + \frac{S_6(a^2 - b^2) + S_7(ab + b^2)}{2\hat{\mu}_m gc^2} + \frac{(a^3 - 2b^3 - 3ab^2)}{8c^3}, \tag{7}
\end{aligned}$$

and

$$\begin{aligned}
a &= \mu_f - \hat{\mu}_m - 2\mu_f \nu_m + 2\hat{\mu}_m \nu_f \\
b &= -\hat{\mu}_m \nu_m + \mu_f \nu_f + 2\hat{\mu}_m \nu_m \nu_f - 2\mu_f \nu_m \nu_f \\
c &= (\hat{\mu}_m - \mu_f)(-\hat{\mu}_m + \mu_f - \hat{\mu}_m \nu_m - 2\mu_f \nu_m + 2\hat{\mu}_m \nu_f + \mu_f \nu_f + 2\hat{\mu}_m \nu_m \nu_f - 2\mu_f \nu_m \nu_f) \\
g &= (2 - 2\nu_m). \tag{8}
\end{aligned}$$

The series S_3 , S_6 , S_7 represent the geometry and spatial distribution of the inclusions. For circular cylindrical fibers, they are given in [12], as follows

$$\begin{aligned}
S_3 &= 0.49247 - 0.47603V_f - 0.02748V_f^2 \\
S_6 &= 0.36844 - 0.14944V_f - 0.27152V_f^2 \\
S_7 &= 0.12346 - 0.32035V_f + 0.23517V_f^2. \tag{9}
\end{aligned}$$

were V_f is the fiber volume fraction and the Lamé constants are given by

$$\begin{aligned}
\hat{\lambda}_m &= \frac{\hat{E}_m \nu_m}{(1 + \nu_m)(1 - 2\nu_m)} \\
\hat{\mu}_m &= \frac{\hat{E}_m}{2(1 + \nu_m)}. \tag{10}
\end{aligned}$$

where \hat{E}_m , ν_m , and μ_m are the relaxation modulus, Poisson's ratio, and shear modulus

of the matrix, E_f , ν_f , and μ_f are the elastic modulus, Poisson's ratio, and shear modulus of the fiber.

The Poisson's ratio of the matrix is constant over time, as confirmed experimentally for a broad frequency or temperature range except near the α and β polymer transitions [25]. The matrix data can be represented by any empirical model. To model a material with transverse isotropy, the following averaging procedure is used

$$\hat{C} = \frac{1}{\pi} \int_m^\pi [\bar{\Theta}^{(4)}][\hat{L}][\bar{\Theta}^{(4)}]^T d\theta \quad (11)$$

where $[\bar{\Theta}^{(4)}]$ is the fourth-order orthogonal rotational tensor (see [24,(1.40)]) representing a rotation θ about the x_1 axis (fiber direction). After completing the integration, the tensor \hat{C} is given by

$$\begin{aligned} \hat{C}_{11} &= \hat{L}_{11} \\ \hat{C}_{12} &= \hat{L}_{12} \\ \hat{C}_{22} &= \frac{3}{4}\hat{L}_{22} + \frac{1}{4}\hat{L}_{23} + \frac{1}{2}\hat{L}_{44} \\ \hat{C}_{23} &= \frac{1}{4}\hat{L}_{22} + \frac{3}{4}\hat{L}_{23} - \frac{1}{2}\hat{L}_{44} \\ \hat{C}_{66}(t) &= \hat{L}_{66} \\ \hat{C}_{44} &= \frac{1}{4}(\hat{L}_{22} - \hat{L}_{23} + 2\hat{L}_{44}) \end{aligned} \quad (12)$$

where the new tensor is the averaged relaxation tensor for a transversely isotropic material. The relation between the relaxation tensor $[\hat{C}]$ and the reduced relaxation matrix $[\hat{Q}]$ for a unidirectional layer can be found by applying plain stress conditions $\sigma_3 = \sigma_4 = \sigma_5 = 0$ to the compliance version of the constitutive equation [26]. As in classical lamination theory (CLT, [26]), the reduced relaxation coefficients are given by

$$\begin{aligned} \hat{Q}_{11} &= \hat{C}_{11} - \frac{\hat{C}_{12}^2}{\hat{C}_{22}} \\ \hat{Q}_{12} &= \hat{C}_{12} \left(1 - \frac{\hat{C}_{23}}{\hat{C}_{22}} \right) \\ \hat{Q}_{22} &= \hat{C}_{22} - \frac{\hat{C}_{23}^2}{\hat{C}_{22}} \\ \hat{Q}_{66} &= \hat{C}_{66} \end{aligned} \quad (13)$$

where the reduced relaxation matrix $[\hat{Q}]$ is symmetric. The plane-stress constitutive relationship for a unidirectional layer [26, (5.35)] can be rotated from the material coordinate system (1,2,6) to any other coordinate orientation using the classical transformation equations of CLT [26]. For a layer reinforced with randomly oriented fibers (such as continuous strand mat, chopped strand mat, etc.), we propose the following reduced relaxation matrix

$$\hat{Q}^{CSM} = \frac{1}{\pi} \int_m^\pi [\Theta][\hat{Q}][\Theta]^T d\theta \quad (14)$$

where the rotation matrix $[\Theta]$ is given in [26]. In (10), the stiffness matrix of a unidirectional layer \hat{Q} is rotated to every angle between 0 and π , added, and then divided by the interval π . This is an averaging procedure that yields the stiffness matrix of a layer with randomly oriented fibers \hat{Q}^{CSM} . After completing the integral, the coefficients of the matrix are given by

$$\begin{aligned} \hat{Q}_{11}^{CSM} = \hat{Q}_{22}^{CSM} &= \frac{3}{8}\hat{Q}_{11} + \frac{1}{4}\hat{Q}_{12} + \frac{3}{8}\hat{Q}_{22} + \frac{1}{2}\hat{Q}_{66} \\ \hat{Q}_{12}^{CSM} = \hat{Q}_{21}^{CSM} &= \frac{1}{8}\hat{Q}_{11} + \frac{3}{4}\hat{Q}_{12} + \frac{1}{8}\hat{Q}_{22} - \frac{1}{2}\hat{Q}_{66} \\ \hat{Q}_{66}^{CSM} &= \frac{1}{8}\hat{Q}_{11} - \frac{1}{4}\hat{Q}_{12} + \frac{1}{8}\hat{Q}_{22} + \frac{1}{2}\hat{Q}_{66} \\ \hat{Q}_{16}^{CSM} = \hat{Q}_{26}^{CSM} &= 0 \end{aligned} \quad (15)$$

Next, a laminated composite material can be represented by applying classical lamination theory, but in the Carson domain. If the laminate is symmetric about the midplane surface, the relation between the stress resultants and strains is

$$\begin{Bmatrix} \hat{N}_x \\ \hat{N}_y \\ \hat{N}_{xy} \end{Bmatrix} = \begin{bmatrix} \hat{A}_{11} & \hat{A}_{12} & 0 \\ & \hat{A}_{22} & 0 \\ sym. & & \hat{A}_{66} \end{bmatrix} \begin{Bmatrix} \hat{\varepsilon}_x \\ \hat{\varepsilon}_y \\ \hat{\gamma}_{xy} \end{Bmatrix} \quad (16)$$

with

$$\hat{A}_{ij} = \sum_{k=1}^{N_L} \hat{Q}_{ij}^{(k)} t^{(k)} \quad (17)$$

where the superscript (k) indicates the layer number, $t^{(k)}$ is the layer thickness and N_L is the number of layers. Working in the Carson domain, the relaxation matrix $[\hat{A}]$ is inverted analytically to obtain the creep compliance matrix

$$[\hat{\alpha}] = [\hat{A}]^{-1} \quad (18)$$

Both, the relaxation and compliance matrices (Eq. 17 and 18) are back-transformed to the time domain by a collocation method, which is explained in detail in [24, App. D]. The method provides N points for each coefficient of the creep compliance matrix $[\alpha(t)]$ ($N = 5$ was used in the examples.) Two additional points can be obtained by using the limit value theorems [24, (7.44)]

$$\begin{aligned} f(0) &= \lim_{s \rightarrow \infty} [sF(s)] \\ f(\infty) &= \lim_{s \rightarrow 0} [sF(s)] \end{aligned} \quad (19)$$

Alternatively, one might simply compute the elastic compliance of the laminate $[\alpha(t=0)]$. The resulting $N + 2$ points can be fitted with the same empirical model used to fit the matrix creep data.

6. Modeling Predictions

With the combined shift factors given in Table 4 and the initial compliances given in Fig. 1-2, it is possible to estimate the creep compliance of the matrix at any time, temperature and RH using (1) or (2). On the power law, $(J')^{-1} = 57.6$ GPa and $n = 0.2246$ for all conditions, while on the Maxwell model, $C = 854.4$ GPa-hr.

The viscoelastic response of the composite was predicted following the procedure outlined in Section 5, in terms of the fiber elastic properties and the estimated matrix creep properties. The predicted composite compliance is compared in Fig. 8 to actual composite experimental data, where it is noted that the model provides good predictive capability. Note that the comparison is good up to 22 hr even though it is based on matrix data obtained over a 4-hr period only. Furthermore, this model is unique in that it predicts all the components of the compliance and relaxation including shear and transverse properties.

7. Conclusions

A procedure to reduce experimental creep data using doubly-shifted superposition was presented. It is notable that identical shift-factors were found when the procedure was applied to creep data for matrix and three different laminates. Therefore, it is proven that only matrix data is required to model long-term compliance of arbitrary laminates. In this context, a novel micro-mechanical/laminate model was presented to predict compliance of arbitrarily laminated composites, which may include random and fabric reinforcement. Experimental matrix creep data is presented and used to predict laminate response. Then, laminate creep data is used to validate the prediction methodology, demonstrating a good predictive capability. Within the scope of this investigation, good agreement was achieved between the experimental data of three different laminates and the proposed model at short and long time response, even when the matrix creep behavior was experimentally characterized only for a short time.

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Table 1. Laminate constituents

Laminate	Resin	Fiber	Fiber Volume Fraction	Nominal thickness (mm)
[90/CSM]4] _s	411-350	UM1810	68%	4.115
[90/45/-45/CSM]3] _s	411-350	TVM3408	44%	7.087
[45/-45/CSM]2] _s	411-350	CM5005	46%	6.058

Table 2. Laminate constituents

Fiber Mat	90 (oz/yd ²)	45 (oz/yd ²)	-45 (oz/yd ²)	CSM (oz/ft ²)
UM1810	18			1.0
TVM3408	16	9	9	0.75
CM5005		25	25	0.50

Table 3. Temperature dependency of shift factors at each environmental condition

Environment	a _T	b _T
12.2 °C and 12% RH (*)	2.6835	1.4842
21.1 °C and 12% RH	1	1
21.1 °C and 80% RH	1	1
65.6 °C and 12% RH	0.4415	0.7835
65.6 °C and 80% RH	0.5462	0.4868

(*) At this temperature there is no practical distinction between 12% RH and 0% RH

Table 4. Combined shift factors at each environmental condition

Environment	a _T a _φ	b _T b _φ
12.2 °C and 12% RH (*)	2.6835	1.4842
21.1 °C and 12% RH	1	1
21.1 °C and 80% RH	0.7460	1.0108
65.6 °C and 12% RH	0.4415	0.7835
65.6 °C and 80% RH	0.4615	0.4112

(*) At this temperature there is no practical distinction between 12% RH and 0% RH

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